PERSONAL EQUATION.

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III.

The Nature and Cause of Personal Equation.

In the preceding section of this study, a number of conditions were set forth that may cause the personal equation to vary. In this I propose to exclude all such, and to consider the personal equation in itself under the simplest conditions. It is evident at once that even so, there will be room for classification. The personal equation differs in the two methods of observation, chronographic and eye and ear, and in the individual applications of each. I shall begin with the chronographic, because from a psychological standpoint it is the simplest; and with the observation of a sudden phenomenon, the precise instant of which cannot be foreknown; for example, the emergence of a star from the dark edge of the moon, or a heliotrope signal.

Identically the same thing has been studied by the physiological psychologists under the name of "reaction-times without signal."[1] A stimulus is received by the sense organ (here the retina); the impulse is conducted to the brain; brain changes result, accom-

[1] Wundt: Physiologische Psychologie, 3te Aufl., II, 287. It is customary in most reaction-time experiments to give warning of the approach of the stimulus to be observed, at a short but variable time before its occurrence.

panied on the psychic side by perception, apperception and volition;[1] a motor impulse starts out along the nerves of the arm; the muscular motion of tapping the key is executed; and the reaction is complete. The whole requires over a quarter of a second.[2] The portion to be credited to each stage is still extremely uncertain, but the sense organ and the muscles have short latent periods, the nervous conduction takes time, and the brain changes are very likely slowest of all. The absolute personal equations for this kind of observing should be 0.30 s or over; and, since no one can observe the event before it happens, the relative should be reduced to a few hundredths of a second.

When the instant of the phenomenon can be foreseen with some exactness, as in transit observations, the process is a little different. The two ways of applying the method (waiting for the bisection, or anticipating it) must further be distinguished. The first corresponds closely to the "reaction-times with signal" of the psychologists, the steady approach to the wire giving most exact warning of the time to record. Reaction-times with a signal, as shown by the little table below, are shorter than those without—in gen-

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[1] Perception, apperception and volition mean here respectively, the first consciousness that a sensation is reaching the sensorium, the clear consciousness of the sensation, and the determination to react to it. For a practiced observer reacting to a stimulus of moderate intensity, the last two stages are more or less overlapped.

[2] Wundt gives values for reaction to the sound of a falling ball, with and without signal, as follows:

	Fall of 5 cm.	Fall of 25 cm.
	s.	s.
Without signal,	0.266	0.253
With signal,	0.175	0.076

The reaction-time for light is in general greater than that for sound. The experiments of Renz (*Astr. Nachrichten*, CXIX, 1888, 145) give values for the disappearance and reappearance of an artificial star varying from 0.40 to 0.60 s as the star declines in brightness.

eral, because the signal gives a chance for focusing the attention, tensing up the muscles, etc., and in particular because, when the stimulus is not too faint, it favors the use of the "motor form of reaction."[1]

In this form the attention is turned not to the stimulus, but with extreme concentration to the execution of the reaction; and there is a corresponding tension in the reacting member. The times with this form are about 0.1 s shorter than when the attention is concentrated on the stimulus, even when that is preceded by a signal, that is, for light, from 0.15 to 0.19 s. Wundt considers that when fully established, reaction after this form is probably an acquired brain reflex. The sensation on its entrance into the general field of consciousness at once releases the motor impulse without the mediation of the will, and the apperception of it takes place while the response to it is in execution. The extra-cerebral stages are as before, and may be assumed to be the same in time, except for the additional muscle tension. I am not able to say how frequent this form of reaction is among astronomers, but it is largely a matter of practice and may be frequent among them.

The observers that anticipate introduce still another element. Instead of reacting to an independent sensory stimulus, they react to a stimulus as affected by judgment—that is to say, they tap the key when they see the star in such a position that they judge from its rate and distance that it will be bisected when the key is tapped. The judgment is of course only semi-conscious, if conscious at all, but nevertheless maintains its character. The effect of this way of using the chronographic method in varying conditions has

[1] Wundt, op. cit. II, 265–267.

already been shown. It certainly adds a rather complex psychological process to those before mentioned, though not one that need make any great difference in accuracy while conditions remain the same.

It is hardly necessary to point out that in all these cases the direct cause of the personal equation is the physiological inertia of the observer. To this the anticipators add a mental factor that compensates partially or wholly, or over-compensates, the physiological delay. Astronomers can therefore hardly hope to free their chronographic observations absolutely from the variations of personal equation. The nearest approach is probably to be found in the use of suitable artificial transits at the time and under the circumstances of the observations to be corrected.

The Eye and Ear Method.

In this method the processes are more complicated than in the last. Instead of executing a definite and predetermined motion in response to a definite sensation, the observer has here to fix disparate sensations with reference to one another—a much more difficult thing to do and one more open to illusion. The greater difficulty is witnessed by the larger probable errors and personal differences that attend this method.[1]

Let me first take up as before the observation of instantaneous phenomena. It is more than likely that in this method, as in the other, foreknowledge of the time at which the phenomenon is to appear, affects the observing of it; but such figures as I have found are insufficient to establish such a difference, and I must therefore speak of both cases together.

[1] American Journal of Psychology, II, 20–21 and 272.

The absolute error in observations of this kind has been measured by Renz with artificial stars.[1] In the preliminary report published the average values are irregular and for the most part do not exceed a few hundredths of a second. Whether this same result would be found with longer series and on other individuals remains to be seen. Determinations of relative personal difference in observing a sudden sound were made by Encke.[2] The mean result of two series was as follows:

$$\begin{aligned} E - B &= +\overset{s.}{0}.261 \\ E - F &= +0.031 \\ E - Q &= +0.083 \end{aligned}$$

The observer B, before knowing the result of the observations, remarked that he felt he had observed much too soon. The mean of two series taken later showed him in the right:

$$\begin{aligned} E - B &= +\overset{s.}{0}.026 \\ E - F &= +0.032 \end{aligned}$$

Later still, observers B and Q found as the mean of three series of comparisons:

$$Q - B = +\overset{s.}{0}.130$$

The mechanism of the observation is easy to understand. The observer receives and counts the regular series of auditory sensations, and into this at some point is shot a sensation of sight.[3] The auditory sensations are practically instantaneous, that of sight leaves an after-image. The senses operate quite as

[1] *Astr. Nachrichten*, CXIX, 1888, 145.

[2] Abhandlungen der Berliner Akademie, 1858.

[3] It makes no great difference whether the observer attends at the instant of observation to a subjective clock-beat, as Wolf thinks, nor whether he counts mentally; that there should be subjective hearing and subjective counting is sufficient.

usual, and we must look elsewhere for the largest part of the error. One can follow the rhythm of the seconds with very great accuracy; it is the entrance of the new stimulus, and, as I hope will appear before the end of the paper, the disturbance of attention thus produced that are really to blame.

The sight stimulus may fall either very near one of the auditory series or in the midst between two, and there is room for some difference in the result according to which happens: if the first, the persistence of the sensations is involved; if the second, the estimate of time. On the first, some light is thrown by experiments of Exner, the Vienna physiologist.[1]

Exner's problem was: How far apart in time must two stimuli be in order to be perceived as successive? The first part of his study deals with stimuli to the same sense; for sight he used electric sparks or little illuminated holes seen one after another through a slit in a revolving disk; and for hearing, the clicks of a lead slip held against a toothed wheel. The next—and these experiments are most like the observations in question—refers to stimuli to different senses. His apparatus furnished a bell stroke and an electric spark, or a shock and a spark, of which the order and separation in time could be varied by an assistant. The observer was required to say whether the sensations were simultaneous, and if not, which came first. The couple were apparently repeated a number of times in the same order before a judgment was made. There are not many experiments reported, and the results are not easy to gather in statistical shape, but they show that "if a sense impression acts simultaneously on both eye and ear, the sensation of hearing

[1] Pflüger's Archiv, Bd. XI, pp. 422–428.

will be perceived earlier than the sensation of sight," and they further give the basis for the second part of the table of approximate times (Exner himself as observer), with which he concludes that section of his article. I extract from his table those values that are of interest in this connection; the double stimuli are supposed to occur in the order given.

	s.
Two noises	0.002
Spark images at the center of the retina	0.044
Bright disks " " " "	0.045
Noises at the two ears	0.064
Hearing—sight	0.06
Sight—hearing	0.16
Touch—sight	0.053
Sight—touch	0.071

In this table it is to be noticed that the time is lengthened when the stimuli reach different senses (or even different sense organs, in the case of the ears); and further, that the times are longer when sight leads, because of the longer after-sensation in that sense, and longest of all when sight leads and hearing follows.[1] The hypothesis that the difference depends largely on the adjustment of attention is supported by Exner's subjective observations. He thus describes the subjective side of the double stimuli experiments:

"A second way of adjusting consists in the following: We adjust for a definite one of the two stimuli, *e. g.* for a bell stroke in connection with the state of the sensorium at the instant of this bell stroke.[2] Then

[1] In experiments with hearing and touch, the perception of succession was much more exact, but the amount of separation necessary did not appear.

[2] In speaking of the first way of adjusting he had said: "We adjust the attention for the first sense stimulus that is to strike us, without knowing which it is, of course. Not for this alone, however, but for the state of the sensorium—I cannot express myself differently—at the instant of the first sense stimulus."

the other stimulus, *e. g.* the electric spark, comes into relation with the first either as preceding or following. One circumstance is here highly disturbing. The impression that is not adjusted for is much weaker in the memory-image than the one adjusted for; it is somewhat indistinct, poorly fixed as to time. There is, therefore, great uncertainty in this method. Usually the inclination is to take the subjectively stronger stimulus, the one adjusted for, for the first, just as the inclination is to take a considerably stronger objective stimulus for the first. Still the whole circumstance can also be reversed; it was in the experiments between touch and sight. There it often seemed to me as if the impression not fixed had already taken place at the time of the fixed; the phenomenon can suddenly change over again into the usual form. A similar phenomenon has been described by Wundt.[1] He states that between sight and hearing he can voluntarily perceive one or the other stimulus earlier according as he turns his attention to them. I have never succeeded with this combination, and the physiological delay of the sight impression comes into consideration, which Wundt had not yet recognized."[2]

[1] Vorlesungen über Menschen- und Thierseele, Leipzig, 1863, I, 39.

[2] In the last edition of his *Physiologische Psychologie*, Wundt makes a similar statement. In speaking of the effect on the simple reaction-time of introducing a stimulus to another sense at almost the same instant as that to be reacted to, he says (II, 294): "Provisionally it may be remarked that the succession of our sense perceptions does not necessarily agree in order with the succession of sense stimuli, but that an actually later impression can readily be anticipated. Introspection leaves no doubt as to the origin of these illusions; they rest on the varying tension of attention. As soon as the tension for the chief sensation has risen to a certain point, it can bring that sensation, even when it actually follows somewhat later than the accompanying stimulus, in spite of that, into the focus of consciousness at the same time or earlier. The greater the attention the more marked will be the difference in time that can be overcome by it." See also II, 339.

It is evident, of course, that so long as a subject adjusted indifferently now for one sensation and now for the other, his observations would show accidental errors only; but if for any reason he habitually attended to one or the other, a constant error might enter and he would show a personal equation.

It may be mentioned in this connection that Encke found a small personal difference in observing the coincidences of the beats of two clocks running at slightly different rates,[1] and Littrow reports experiments of Weiss's to similar effect.[2]

All this, however, applies only when the phenomenon to be observed falls near one of the second beats. When it falls in the middle of the second, another service is required of the mind, one that probably introduces another error, namely, the estimation of the fractional parts of the second. Unfortunately there are no experimental determinations, so far as I am aware, of this factor. The time-sense has been studied somewhat, but with a different end in view. It may be conjectured that here also attention plays an important role. The grounds of this conjecture and of what has already been said about the function of attention will be somewhat strengthened by what is to be said in the next section, though the reference is not direct enough to justify repetition.

Transit Observations by the Eye and Ear Method.

Much more important, from an astronomical point of view, than the last, is the application of the eye and ear method to the observation of transits. The opera-

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[1]Abhandlungen der Berliner Akademie, 1858.

[2]*Astr. Nachrichten*, LXVIII, 1867, 369.

tions are here very different from those just considered, and more uniform in results, though still attended by considerable personal differences. The effort is to fix two points in the continuous line of motion of the star, one after and one before the crossing of the wire, which shall correspond with successive beats of the clock, to estimate the position of the wire between the two in tenths of the whole space, and thus finally to arrive at the time of the crossing in tenths of a second. The stage at which the important personal differences seem to enter is the connecting of the clock-beat with the position of the star.

The first and chief theory put forward in explanation of these is that of Bessel, given in his original study.[1] He says:

"These different experiments show that no observer, even if he believes he follows Bradley's method of observation in all strictness, can be sure to tell the *absolute* instant of time correctly. If it is assumed that impressions on the eye and the ear cannot be compared with each other in an instant, and that two observers use *different* times for the carrying over of the one impression upon the other, a difference originates—yet a greater, indeed, if one goes over from seeing to hearing, the other from hearing to seeing. That different kinds of observation should be able to alter this difference need not seem strange, if one assumes as probable that an impression on one of two senses *alone* will be perceived either quite or nearly in the same instant that it happens, and that only the entrance of a second impression produces a disturb-

[1] Königsberg Observations, Abth. VIII, 1822, iii; also his Abhandlungen, Bd. III, p. 303.

ance which varies according to the differing nature of the latter. It is probable that Maskelyne would have agreed more nearly with me than with Struve, Walbeck or Argelander, since in the opposite case a difference of about two seconds would have occurred between Kinnebrook and me, which is surely too great to be considered possible. Maskelyne and I therefore probably follow the custom of going over from hearing the second-beat to seeing, while the astronomers observing later appear to have accepted the contrary."[1]

Bessel's theory was suggested no doubt by his own mental processes in observing, and probably has commended itself to astronomers for similar reasons. Hartmann noticed something of the kind in his experiments,[2] and C. Wolf likewise, at least in his earlier ones, though he had never perceived it in actual observations. "I have been able to establish the existence of this dead time very easily," he says; "I listened to

[1] Faye illustrates the same theory as follows (Comptes rendus, LIX (1864) 475): "Imagine a moment," says he, "that the mind is an eye placed inside the brain, an eye attentive to the modifications that each sensation determines in the nervous tracts, that end there. If sensations of the same nature are produced at the same point, this inner eye will easily judge whether they are successive or simultaneous; but if they arise from different senses whose nerves end in different regions of the brain, the inner eye will have to move in order to pass from one region to another, and the time thus employed will not be perceived; sensations separated by a very real interval will be noted falsely as simultaneous. The time lost, the time thus employed in going from one sensation to the other, can amount to more than a second. It will vary from one individual to another according to the rapidity with which his inner eye is moved to contemplate successively the keys of that prodigiously complex keyboard that we call the brain.

"I do not need to say that I attach no reality to this comparison; our mind is not an inner eye. Nevertheless, the necessity of comparing two sensations of different origin condemns the mind to a peculiar labor, since it uses a time so considerable to establish a communication between different nervous tracts. This task is also very fatiguing, while the comparison of sensations of the same origin is not so, or is a good deal less so."

[2] AMER. JOUR. PSYCHOLOGY, II, 29.

the second, and when the sound had been perceived I brought my attention to the position occupied by the star."

Of similar import is a series of experiments by Wundt,[1] in which the observer tried to fix the position of a pointer (sweeping over a divided arc) which corresponded to the production of a sound or other stimulus. This is hard to do, and it is necessary to allow the sound to come a number of times in succession before any confident judgment can be made, and even of the results so obtained a very large number are necessary to average out accidental variations. It is easy, for example, to unite the sound with a predetermined position of the pointer, if not too far from the true one; and if all the scale is covered except the single division, the error may amount to a quarter second.

At moderate velocities the sound is regularly associated with too early a position. As the rate of both pointer and recurring sounds is increased, the error lessens and becomes zero—for Wundt himself, when the divisions of the scale are passed over in about $\frac{1}{36}$ second each and the sounds are a second apart. If the rate is still further increased the error changes sign, that is, the sound is connected with a position later than the true one. This, however, cannot go far, because the divisions are soon swept over too rapidly to be distinctly seen.[2]

The error changes in the same way also when, instead of a single sound, a sound and an electric

[1] Physiologische Psychologie, II, 334 ff.

[2] Relative personal equations would appear whenever individuals differed in the results obtained in these experiments. One of Wundt's subjects in experiments of this kind always fixed a position of the pointer before the real one.

shock, or a sound and another sound of a different kind, are produced together. The two are perceived as simultaneous, but both are associated with a position of the pointer less behind its true position. A third sensation added changes the sign of the error, and a fourth carries it still further in the same direction. The little table below gives the amount of the error (the average at three rates) for four additions; those marked minus are too early, those plus are too late:

First added stimulus	—0.0670
Second " "	—0.0113
Third " "	+0.0296
Fourth " "	+0.0399

I pass by developments of these experiments, not immediately concerned with the point in question, to notice Wundt's interpretation of his results. It is something as follows. The division of the scale with which the sound shall be connected depends within certain limits on the adjustment of attention alone,[1] and this is in turn influenced by the rate at which the sounds succeed one another and the pointer moves over the scale. When the rate is such that attention is just at its maximum when the sound is each time produced, there is no error; the sound is associated with the true position of the pointer. When the rate is slower the adjustment takes place too soon and the sound is associated with a position earlier than the true one. When the rate is faster the adjustment is too late and the position later than the true one. The speed at which the pointer moves also influences the rate of adjustment; as the pointer moves fast or slow

[1] The attention may be thought of as rising rhythmically again and again like a wave, the time at which it reaches its maximum fixing the time at which the sensation to which it is directed will be perceived.

the adjustment is made rapidly or slowly. When other stimuli are sent in at the same time as the sound, the adjustment becomes more difficult and the time required for it longer; hence the tendency of the first error to grow less and finally to change sign.

Wundt's "adjustment of attention" is only the psychological explanation of Bessel's indefinite "carrying over of one sensation upon another." Both are agreed in putting the cause of personal equation in the difficulty the mind finds in uniting disparate sensations. C. Wolfe,[1] however, believes that, when such errors as these have been removed by training, there still remains an error due to purely physiological causes. To the examination of this theory and the experiments upon which it rests I turn next.

Wolf clears the ground for the discussion by collecting a large number of personal differences and showing that those larger than 0.3 s are rare. At the beginning of his experiments his own absolute personal equation was of that amount, but as they continued it fell rapidly to 0.11 s, and there remained fairly constant as if it had reached a physiological limit. In the instant of observation an observer does not hear the actual clock-beats, but follows an inner series; he may even go on to complete an observation when the real beats have ceased, without knowing the difference. If this is so, what becomes of the superposition of dissimilar sensations? And again, the seconds need not be taken by ear, but by touch or even by sight, and yet the personal equation remains practically the same.

These last are interesting observations, but it must

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[1] Annales de l'observatoire de Paris; Mémoires, VIII, 185 ff.

be perfectly clear that they have no force against Bessel. The illusion of a clock-beat is of its very nature the same to the mind as a real one. When the second is taken by touch there is a dissimilar sensation to be united with that of sight as much as when the second is heard, and even where it is taken by sight there are two things to be attended to, the time and the position of the star, to say nothing of the incipient muscular discharges that probably accompany even subjective counting.

Wolf's theory, however, does not fall with these general arguments; its corner-stone is the following set of experiments. Making use again of his artificial transits, he had, instead of the single little illuminated hole that gave him his star before, three little holes close together in the same vertical line. The upper and lower were illuminated at intervals by induction sparks, the middle one constantly. As long as the plate containing the holes was still, all three seemed, as they really were, in the same vertical line. When the plate was in motion, the constantly illuminated star seemed in advance of the sparks whichever the direction of motion, and further in advance as the rate was more rapid. If the sparks flashed irregularly the star was clear cut; if they flashed regularly, say every second, so that they might be anticipated, the star, though as far ahead as before, had behind it a train extending backward as far as the line connecting the sparks. If the star was made to disappear at the instant the sparks flashed, it was seen as at first on the same line with them.

The interpretation of these experiments is so important to Wolf's theory that I give his own words:

"These experiments seem to me to contain all the

facts relative to the personal equation in the case where the second is perceived by sight, and to give the explanation of it. In the first place, the last shows us that the deviation of the star is a pure illusion. The first [*i. e.* that in which the sparks flash irregularly and the star is seen ahead of the sparks without a train] represents very nearly the phenomenon cited by M. Faye to explain the ordinary error of observation. At the unforeseen moment when the sparks flash, the eye, surprised by their apparition, turns its whole attention to them, ceases to see the star (which continues its movement), and does not return to it before the sparks have disappeared. The star has then advanced, and appears in advance by an amount *equivalent to the time during which the luminous impression of the two sparks persists.* There is then, to be sure, between the moment when the second is perceived and the moment when the eye registers the position of the star, a lost time; but in place of attributing, with M. Faye, a purely psychological cause to this lost time, I find for it a physiological origin and a value physiologically determined. The error committed in the estimate of the position of the star is equal to the duration of the persistence of the luminous impression.

"But this case, I have said, is not really that of an experienced observer that perceives the sensation of a perfectly rhythmed second. It is necessary then to go to the second experiment [*i. e.* that in which the sparks flash regularly and the star is seen in advance with a train]. This time the eye, forewarned of the instant of the apparition of the sparks, does not cease to see the star, and at the instant of the explosion sees it on the line of the sparks. But it continues to see them

during a certain time during which the star advances and reaches the same position as before at the instant of the disappearance of the sparks. Thus the eye sees the star at once in all the positions comprised between the two extremes that I have just defined. For it these positions are simultaneous; their extent corresponds then to an indivisible space of time. And consequently the observer can relate the position of the star at the moment when the sparks have flashed to any one of the points of that extent.

"But it is necessary to remark besides that at the moment when the eye begins to perceive the sparks, it has still the sensation of all the positions occupied by the star before, during an interval corresponding likewise to the duration of the persistence of the luminous impression. The observer can, therefore, also relate the position of the star at the moment when he perceives the second to any one of the points of this previous extent.

"I will conclude, then, from this analysis, that the personal correction of an observer that perceives by sight an exactly rhythmed second is necessarily comprised between two limits, which are the duration of the persistence of the luminous impression taken positively and negatively."[1]

The next step is to extend the explanation to the case where the time is taken by other senses.

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[1] What he seems to mean can perhaps be shown by the figure below. The lines represent the central star with its train, the dots the occurrence of the sparks. The motion is supposed to be from right to left.

a *b*

The state of things in experiment two is represented at *a* ; any position of the star in the line may be associated with the instant of the sparks. At *b* is represented the state of things in the next to the last paragraph ; as before, any position of the star in the line may be associated with the instant of the sparks.

"I will remark," says Wolf:

"1st. That according to the experiments that I have cited above, my personal correction remains the same in whatever manner the perception of the second reaches me, by ear, by sight, or by touch;

"2d. That the duration of the auditory sensation being, according to the physiologists, less than 0.01 s, this duration cannot enter into the cause of personal equation, or at least cannot make it vary except in amounts less than those that we measure. . . .

"It follows from this that the cause of the personal error ought to be here the same as in the first case, that is to say, it is still found in the combination of the persistence of the luminous impression with the continuous movement of the star."

The author then repeats in substance the explanation of the positive and negative limits given above, and restates the conclusion for the case of eye and ear observations.

Wolf shows, earlier in the paper, that when the apparent motion of the star is suppressed by illuminating it with a spark from second to second, the personal equation practically disappears, and now adds a last item of evidence in showing that the duration of the after-image under conditions like those of his experiments was from 0.05 s, when the stimuli fell continuously on one point of the retina, to 0.16 s, when the image moved upon the retina. The absolute personal equation, of the kind in question, therefore should not exceed 0.16 s. In case the eye is not stationary (as has been supposed in the discussion, and as Wolf believed his own to be during the time that the star was near the wire) the explanation is still applicable; for if the eye followed the star, the images of

the wires would move upon the retina, and the after-sensations left by them would bring about the error.

I have quoted from Wolf's interpretation above because I propose to criticise it; I pass by minor matters for his cardinal experiment, experiment two. Four things contribute to this illusion: (1) the persistence of the images of the sparks; (2) the persistence of the image of the star; (3) its motion, and (4) the turning of a portion of the attention to the sparks. Assuming Wolf to be correct in implying that the attention or a part of it is turned to the sparks and remains there till the after-images have disappeared, it is clear that if the images of the sparks had no persistence, the middle point could not move during their persistence, nor be seen in advance of them; its train therefore depends on their persistence. In his further discussion Wolf seems to forget this essential point and speaks only of the train, that is, of persistence of the star's after-image. This omission lets in two errors. In the first place, if the train of star after-images depends on the persistence of the after-images of the sparks, there can be no train before the sparks flash, and consequently no associating the flash with a position of the star behind the true one, that is, no minus personal corrections. And second, if the persistence of the spark-images is essential to the illusion, it is wrong to think that the illusion is the same when the sparks are replaced by a sound of which the after-sensation is confessedly less than 0.01 s. It is, of course, easy to avoid these difficulties by making the time given to the sparks due to the inertia of attention and not to the persistence of their after-images, but that would be to surrender the point for which Wolf is contending. At the same time it is not to be denied

a priori that physiological inertia may enter to a certain extent. The fact that the eye and ear personal equation is larger for bright stars than for faint ones, as proved by Bakhuyzen,[1] could be cited in support of such a view. On the whole it may be admitted that Wolf has brought to notice a possible factor in personal equation, but it cannot be admitted that he has demonstrated the large place for it that he claimed.

Since the beginning of the year, another physiological explanation has been proposed by J. J. Landerer.[2] He finds that with very many people, himself included, there is a monocular doubling of the image of a single bright point seen at the distance of distinct vision or a little beyond, the second image being a little fainter than the original. This doubling in many is from left to right in the right eye, and from right to lèft in the left eye, though in some it is vertical or inclines, and in one individual was not found at all. Even when observed with a lens, the double partly overlies the principal image. In view of this, he believes "that the observation of a little luminous disk is made not on its center, but to the right or left, above or below, according to the direction of the diplopia, the eye quite naturally having regard to the center of the group that the two images form. It follows then that the efficient cause of personal equation properly so called rests in this physiological effect, or at least that this plays a preponderant role." If this is so, it is easy to calculate the personal equation for different powers of eye-piece and different declinations,

[1] See AMERICAN JOURNAL OF PSYCHOLOGY, II, 278.

[2] Comptes rendus, CVIII, 219; Feb. 4, 1889.

and in the author's case the values so found agree within a few hundredths of a second with those found in the ordinary way.

It is impossible to deny off-hand that such optical irregularities do affect observation, but after all that has been adduced in evidence of the general complexity of personal equation and the probable importance of the psychic factor in it, I cannot agree in the important rank that the author assigns to them. There is also some reason to think that some at least (in using the chronographic method) observe rather the advancing limb of the star's image than the center.

Bredichin[1] incidentally suggests a special illusion in eye and ear observations—a *sui generis* persistence of apparent motion, arising from the constant decreasing of the distance between the star and wire before the transit and the constant increasing of the distance after the transit. This would of course only fit the case of those that observe too early. Such a thing is an interesting possibility, but so far as I know there is as yet no evidence for it, except so far as it might rest on the play of attention already many times mentioned.

APPENDIX.

I add this appendix chiefly for two reasons. The first is that I wish to thank Professors Eastman and Winlock and Mr. A. S. Flint of the Naval Observatory for courtesies shown me at that institution. The second is that I may give what bibliographical references I have gathered. The list is not exhaustive. I have rather set down such references as it has seemed to me, either from having seen the articles themselves or from some other reason, might be convenient in preparing such a general sketch as has been presented. As that was confined to the form of personal equation that attends time observations, so no references are here given to any other form. The list can be enlarged somewhat, if any one desires, from Houzeau and Lancaster's *Bibliographie générale de l'Astronomie*, Vol. II, 902 ff. and

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[1]Annales de l'observatoire de Moscou, II, 2nd pt., p. 69.

1685, and from Sinclair's *General Index of Scientific Papers contained in the Appendices of the Reports of the U. S. Coast Survey:* Report for 1881, app. 6, pp. 91-123. Figures for the relative personal equation, at least, can also be found in the reports of almost every determination of longitude and in the transit observations of the different observatories.

BIBLIOGRAPHY.

AIRY: [*Personal equation in observing the limb of the Moon.*] Greenwich Astronomical Observations, 1848, p. lxx.

—— [*Relative personal equations with the binocular eye-piece.*] Greenwich Astronomical Observations, 1852, p. xlvi, and 1853, p. xliv.

—— [*On C. Braun's priority in suggesting the transit-micrometer.*] Monthly Notices, XXV, 1865, 157.

ARAGO: *Sur un moyen très-simple de s'affranchir des erreurs personnelles dans les observations des passages des astres au méridien.* Comptes rendus, XXXVI, 1853, 276. Also Œuvres d'Arago, XI, 233; in abstract, Cosmos, II, 1853, 348.

ARGELANDER: [*Effect of attention on eye and ear observations.*] Tageblatt der 36ten Versammlung deutscher Naturforscher und Aerzte in Speyer, 1861, p. 25.

—— [*On the observation of faint stars.*] Bonner Beobachtungen, VI, 1867, p. xi.

—— *Verschiedenartige Auffassung des Momentes des Durchganges durch den Faden bei schwachen und hellen Sternen.* Astr. Nachrichten, LXXIV, 1869, 263.

—— [*On relative inaccuracy of transits over the first wire of the reticle.*] Vierteljahrsschrift d. astr. Gesellschaft, 1872, p. 16.

BACHE and SCHOTT: *Notice of the determination of the longitude of Fernandia* . . . . U. S. Coast Survey Report for 1857, Appendix 30, p. 316. Also Proc. Amer. Assoc., Montreal meeting, XI, 166.

BAKHUYZEN: *Die Aenderung des persönlichen Fehlers bei der Beobachtung von Sternen verschiedener Helligkeit.* Astr. Nachrichten, XCV, 1879, 81.

—— *Beobachtungen zur Bestimmung des persönlichen Durchgangsfehlers mit der Helligkeit der Sterne.* Vierteljahrsschrift d. astr. Gesellschaft, XIV, 1879, 408.

—— *Apparat zur Bestimmung des absoluten persönlichen Fehlers bei Durchgangs-Beobachtungen an Meridianinstrumenten.* Vierteljahrsschrift d. astr. Gesellschaft, XIV, 1879, 414.

BESSEL: *Persönliche Gleichung bei Durchgangsbeobachtungen.* Astronomische Beobachtungen der Sternwarte zu Königsberg. Abth. VIII (1822), iii; additional notes, Abth. XI, iv, and Abth. XVIII, iii. Also in Bessel's Abhandlungen, Bd. III, pp. 300–304.

BOND, G. P. and R. F.: *Description of an apparatus for making astronomical observations by means of electro-magnetism.* Report Brit. Association, Ipswich meeting, 1851, page 21 of abstracts.

BRAUN: *Das Passagen-Micrometer.* *Leipzig*, 1865.

BRAUN: *Das Passagen-Micrometer.* Berichte von dem Erzbischöflich-Haynald'schen Observatorium zu Kalocsa, 1886, p. 163.

BREDICHIN: *Sur l'équation personnelle absolue.* Annales de l'Observatoire de Moscou, II, pt. 2, 1876, 69.

BROWNELL: *Does it take time to think?* Popular Science Monthly, XI, 1877, 564.
Popular treatment of personal equation and reaction times.

BUCCOLA: *La legge del tempo. Milan*, 1883.
Especially pp. 27–29 and 160–163.

CAMPBELL: *On a peculiarity of personal equation.* Monthly Notices, XXXVII, 1877, 283.

CRISTIE: *Description of the personal equation machine of the Royal Observatory, Greenwich.* Monthly Notices, XLVIII, 1887, 1.

DOWNING: *On the probable errors of transits of the first and second limbs of the Sun, observed by the chronographic method.* Monthly Notices, XXXVIII, 1878, 102.

DREYER: *On personal errors in astronomical transit observations.* Proc. of Irish Acad., 2d Series, II (1877), 484; abstract, Vierteljahrsschrift d. astr. Gesellschaft, XII, 1877, 246.

DUNKIN: *Comparison of the probable error of a transit of a star observed with the transit-circle by the "eye and ear" and chronographic methods.* Monthly Notices, XX, 1860, 86.

—— *On the probable error of a meridianal transit-observation, by the eye and ear and chronographic methods.* Monthly Notices, XXIV, 1864, 152.

—— *On personality in observing transits of the limb of the Moon.* Monthly Notices, XXIX, 1869, 259.

—— *On personality in observing transits of the first and second limbs of the Sun.* Monthly Notices, XXXV, 1875, 91.

—— *On the effects of personality on the tabular errors of the Moon.* Monthly Notices, XL, 1880, 165.

EASTMAN: *Personal equation apparatus.* Astronomical Observations at the U. S. Naval Observatory, 1875, p. xxi.

ENCKE: *Ueber die Längenbestimmung von Berlin und Königsberg mittelst des Telegraphen.* Monatsberichte der Akad. Berlin, 1857, pp. 613–618.

—— *Ueber die Bestimmung des Längen-Unterschiedes zwischen .... Brüssel und Berlin*, 1857. Abhandlungen der Berliner Akademie, 1858.
Translated by Quetelet in the Annales de l'Observatoire de Bruxelles, XIII.

EXNER: *Experimentelle Untersuchung der einfachsten psychischen Processe.* Archiv für die ges. Physiologie (Pflüger's), Bd. VII, 601, and Bd. XI, 403. See also his section on personal equation in Hermann's Handbuch der Physiologie, II, 255–277.

FAYE: *Sur les erreurs d'origine physiologique.* Comptes rendus, LIX, 1864, 473. For his remarks on transit observations by photography see Comptes rendus, XXVIII, 1849, 241; XLVI, 1858, 705; L, 1860, 965.

FOERSTER: *Uebersicht der neueren Arbeiten betreffend die Ermittelung der persönlichen Gleichung und ihre Aufhebung bei Zeitbeobachtungen.* Vierteljahrsschrift d. astr. Gesellschaft, I, 1866, 236.
General review of work of Hirsch and Plantamour, C. Wolf, Braun, and Hartmann.

GERLING: *Die Längenunterschiede zwischen Göttingen (Altona), Marburg und Mannheim durch Signale bestimmt.* Astr. Nachrichten, XV, 1838, 249.

GILL: *On the results of meridian observations of the Mars comparison stars.* Monthly Notices, XXXIX, 1878, 98.
Gives Bakhuyzen's curves for the effect of brightness.

—— *On the value of the solar parallax derived from observations of Mars made at Ascension Island during the opposition of* 1877. Monthly Notices, XXXIX, 1879, 434.

GONNESSIAT: *Sur quelques erreurs affectant les observations de passages.* Comptes rendus, CVII, 1888, 647.

GOUJON: *Sur la détermination du diamètre du soleil par les observations faites à la lunette méridienne.* Comptes rendus, XXVIII, 1849, 220.

GOULD: *Abstract of the report . . . on the determination of the longitude of Albany, N. Y., by the telegraphic method.* U. S. Coast Survey Report for 1861, Appendix 18, pp. 228–229.

—— *On the longitude between America and Europe from signals through the Atlantic cable.* U. S. Coast Survey Report for 1867, Appendix 6, pp. 104–112.
Good on the causes of variation of personal equation.

HARKNESS: [*Personal equation machine.*] Astronomical Observations, U. S. Naval Observatory, 1867, Appendix 1, p. 13.

HARTMANN: *Einige Beobachtungen und Bemerkungen über Personaldifferenz.* Archiv der Math. u. Phys. (Grunert's), XXXI, 1858, 1; slightly abbreviated, Astr. Nachrichten, LXV, 1865, 129.

HILFIKER: *Ueber eine persönliche Gleichung bei Durchgangsbeobachtungen.* Astr. Nachrichten, CXVIII, 1887, 99. On the changeability of the difference of personal equation by the two methods. See also review in "The Observatory," XI, 1888, 95, where corresponding figures are given from the Greenwich records.

—— *Sur l'équation personnelle dans les observations de passage.* Bulletin de la Soc. de Sc. Nat. de Neuchâtel (séance du 14 Avril 1887), XVI, 1888, p. 47.

HILGARD: *Description of two forms of portable apparatus for the determination of personal equation* . . . . . U. S. Coast Survey Report for 1874, Appendix 17, pp. 156–162. The first and inferior form is that described in the Comptes rendus, LXXIX, 1874, 999.

HIRSCH: *Sur les corrections et équations personnelles dans les observations chronographiques de passage.* Bulletin de la Soc. de Sci. nat. de Neuchâtel, VI, 1863, 365. Also Moleschott's Untersuchungen zur Naturlehre der Menschen und Thiere, IX, 200.

—— *Sur quelques recherches récentes concernant l'équation personnelle et le temps physiologique.* Bul. de la Soc. de Sci. nat. de Neuchâtel, X, 1876, Appendix.
On Exner's paper in Pflüger's Archiv, VII, 601.

HIRSCH and PLANTAMOUR: *Détermination télégraphique de la différence de longitude entre . . . Genève et Neuchâtel.* Genève et Bâle, 1864.
Much the same as elsewhere reported by Hirsch.

——, WOLF, R., and PLANTAMOUR: *Détermination télégraphique de la différence de longitude entre Righi, Neuchâtel et Zurich.*
Strong on R. Wolf's studies on personal equation.

HOLDEN: *The personal equation.* Popular Science Monthly, VI, 1875, 385.
Popular treatment of the subject.

—— [*The personal equation.*] Practical paper No. 5, U. S. Hydrographic Office.

—— *Determination of the effect of the magnitude of a star on the observed time of its transit by means of wire screens.* Astr. Nachrichten, CXIII, 1885, 17.
Suggests a precaution to be used in such cases.

HOUGH: *Remarks on personal equation in transit observations.* Proceedings Amer. Assoc. XVI, 1867, 22.

KAISER, F.: *Ueber eine neue Anwendung vom Princip der Noniën zur genaueren Beobachtung plötzlicher Erscheinungen.* Tijdschrift voor de Wis- en Natuurkundige Wetenschappen, Vol. V, 1851.

—— *De volledige bepaling van persoonlijke fouten bij sterrekundige waarnemingen.* Verslagen en mededeelingen der Akademie der wetenschappen, Amsterdam, Afdeeling natuurkunde, XV, 1863, 173. Also in extract in the Archives néerlandaises des sciences, Harlem, I, 1866, 193.

—— *Ueber einen neuen Apparat zur absoluten Bestimmung von persönlichen Fehlern bei astronomischen Beobachtungen.* Verslagen en mededeelingen der Akademie der wetenschappen, Amsterdam, Afdeeling natuurkunde, 2de Reeks, Deel II, 1868, 216.

—— *Beschreibung der Zeitcollimatoren der Sternwarte in Leiden.* Annalen der Sternwarte in Leiden, II, 1870, [19].

KAYSER, E.: *Ein Mittel den persönlichen Fehler bei Passagen-Beobachtungen zu bestimmen.* Schriften der Naturforsch. Gesells. in Danzig, neue Folge, II, 1868, 1. Author's abstract, Astr. Nachrichten, LXX, 1868, 129.

KÜSTNER: *Resultate der Beobachtungen von* 670 *Sternen, angestellt am grossen Berliner Meridiankreise.* Berlin, 1887. (Astronom. Beobachtungen auf der kgl. Sternwarte zu Berlin.)

LADD: *Physiological psychology*, pp. 468–497. Résumé of reaction-times, etc., in general.

LANDERER: *Sur l'équation personnelle.* Comptes rendus, CVIII, 1889, 219.

LANGLEY: *On the possibility of transit observations without personal error.* Amer. Jour. Science, 3d series, XIV, 1877, 55.

LEITZMANN: *Ueber Störungserscheinungen bei astronomischer Registrirung.* Wundt's Philos. Studien, V, 56.

LIAIS: *Sur un procédé substituer des opérations de pointé aux estimations de passages dans les observations astronomiques azimuthales.* (Extract by the author.) Comptes rendus, XLVI, 1858, 131.

LIAIS: *Sur les équations personnelles et les moyens de les faire disparaître.* Mém. de la Soc. des Sciences nat. de Cherbourg, VI, 1858, 218.

v. LITTROW: *Beiträge zur Kenntniss der persönlichen Gleichungen.* Astr. Nachrichten, LXVIII, 1867, 369. Also in greater detail in v. Littrow's work, Bestimmung der Meridiandifferenz Leipzig—Dablitz, 1868.

MASKELYNE: [*On Kinnebrook's observing.*] Greenwich Astronomical Observations, III, 1795, 319, 339.

MAURY: [*On a method of comparing for personal equation.*] Astronomical Observations at the U. S. Naval Observatory, I, 1845, xlix.

MITCHEL: *On personal equation.* Monthly Notices, XVIII, 1858, 261; also Journal of the Franklin Institute, LXVI, 1858, 349.

NEWCOMB: [*Effect of magnitude on personal equation.*] Observations at the U. S. Naval Observatory, XIV, 1867, Appendix 3, p. 27.

NEISON: *On the correction for personality required by the observations of the Moon made with the Greenwich transit-circle.* Monthly Notices, XL, 1880, 75.

—— *On the determination of the personal equation of lunar observers.* Monthly Notices, XL, 1880, 302.

NICOLAI: *Ueber die bei den einzelnen Individuen stattfindende Verschiedenheit des geistigen Reflexes der äussern Eindrücke auf die Organe des Gesichts und Gehörs.* Isis, XXIII, 1830, 678.

PAPE: *Ueber die Genauigkeit der Beobachtungen der Rectascensionen bei Anwendung chronographischer Apparats.* Astr. Nachrichten, LIV, 1860, 177.

PEIRCE, B.: *Abstract of a memoir on the personal peculiarities of astronomical observers.* Proceedings of the American Academy, Boston, IV, 1860, 197.

PETERS: *Beschreibung eines auf der Altonaer Sternwarte aufgestellten galvanischen Registrirapparats für Durchgangs-Beobachtungen, nebst Vergleichung einiger an denselben bestimmten Personal-Differenzen mit solchen, die auf gewönliche Weise gefunden sind.* Astr. Nachrichten, XLIX, 1859, 1. Also, somewhat changed, in Peter's work, Ueber die Bestimmung des Längenunterschiedes zwischen Altona und Schwerin, Altona, 1861.

An important paper, especially for history of personal equation.

PLANTAMOUR and LÖW: *Détermination télégraphique de la différence de longitude entre Genève et Strasbourg*, 1876.

PRAZMOWSKI: *Erreurs personnelles qui affectent les passages des astres . . . . moyens de les éliminer.* Cosmos, IV, 1854, 545.

—— [*Artificial transit apparatus.*] Comptes rendus, XXXVIII, 1854, 748. A very brief note.

QUETELET: *Détermination des équations personnelles des observateurs.* Nouveaux mém. de l'Académie, Bruxelles, XVI, 1843, No. 1, p. 4.

RADAU: *Sur les erreurs personnelles.* Le Moniteur sc. du chim. et du manufac. (Quesneville), VII, 1865, 977; VIII, 1866, 97 et al.;

IX, 1867, 416. Also Repertorium für physikal. Technik u. s. w. (Carl's), I, 1866, 202, 306; II, 1867, 115; IV, 1868, 147.

A very full and excellent general account of the subject to its date; freely used in the preparation of these papers, especially the first.

RADIER: [*Device for excluding personal equation in observing.*] Les Mondes, I, and The Reader, IV, 1864, 141.

RAGOONATHA CHAREY: *On the determination of personal equation by observations of the projected image of the Sun.* Monthly Notices, XIX, 1859, 338.

RENZ: *Versuch einer Bestimmung der persönlichen Gleichung bei der Beobachtung von Sternbedeckungen.* Astr. Nachrichten, CXIX, 1888, 145.

REPSOLD: *Durchgangsinstrument mit Uhrbewegung.* Astr. Nachrichten, CXVIII, 1888, 305. Noted in "The Observatory," July, 1888.

ROBINSON: *Probable errors of eye and ear in transit observations.* Proceedings of Royal Irish Academy, V, 1853, 366.

—— [*Personal equation for limbs of the Moon.*] Armagh Catalogue, 1859, Introduction, p. x.

ROGERS: *On the variability of personal equation in transit observations.* American Journal of Science, 2d series, XLVII, 1869, 297.

SAEGMÜLLER: [*Prismatic double eye-piece for observing to determine relative personal equation.*] Transactions of the International Geodetic Association for 1878.

SCHOTT: *Determination of longitude by means of the electric telegraph.* Report of Supt. U. S. Coast Survey for 1880, Appendix 14, pp. 233-234.

SHEEPSHANKS: *Proposal of a method for eliminating the effect of personal equation in computing longitudes from transits of the Moon.* Monthly Notices, IX, 1849, 142.

STRUVE: *Détermination de la longitude entre Pulkova et Altona*, 1843. See also *Expéditions chronométriques de* 1845, and *Mémoires de l'Observatoire central*, II, 1859.

—— [*Variation of personal equation with brightness.*] Positiones mediae, 1852, p. lxiv.

TACCHINI: *Sulla equazione personale.* Rivista sicula di scienze, lettere ed arti, Anno I, vol. 2°.

TURNER: *Results obtained with the personal equation machine at Greenwich.* Monthly Notices, XLVIII, 1887, 4. Discussed also in "The Observatory," X, 1887, 408.

WAGNER: *Ueber die Abhängigkeit der persönlichen Gleichung von den Declinationen der Sterne.* Tageblatt der 49ten Versammlung deutscher Naturforscher und Aerzte in Hamburg, 1876, Beilage, p. 62.

—— [*On the influence of rate and direction of motion on personal equation.*] Observations de Poulkova, XII, 1887.

WISLICENUS: *Untersuchungen über den absoluten persönlichen Fehler bei Durchgangsbeobachtungen*, pp. 50. Leipzig, 1888. Abstract in Amer. Jour. Sci., 3d series, XXXVII, 1889, 116.

WOLF, C.: *Recherches sur l'équation personnelle dans les observations de passages, sa détermination absolue, ses lois et son origine.* Annales de l'Observatoire de Paris, Mémoires, VIII, 1866, 153. Author's abstract, Comptes rendus, LX, 1865, 1268.

WOLF, R.: *Ueber die sogenannte Personaldifferenz.* Verhandl. der Schweitzer. naturforsch. Gesells. 1869, 85.

—— *Untersuchungen über persönliche Fehler bei Durchgangsbeobachtungen.* Astr. Nachrichten, LXXV, 1870, 71.

—— *Bestimmung der Fadendistanzen am Kern'schen Meridiankreise, und Untersuchung einer eigenthümlichen Anomalie, die bei Ermittelung der Personalgleichung eintreten kann.* Wolf's Mittheilungen, III, 1872, Nos. 25, p. 159, and 26, p. 215. Author's abstract in Astr. Nachrichten, LXXVI, 1870, 369.

—— *Ueber eine neue Methode die Personalgleichung und, wenigstens annährend, die absolute Personalcorrection zu bestimmen.* Vierteljahrsschrift d. naturforsch. Gesells. (Zürich), XXI, 1876, 72; also Wolf's Mittheilungen, IV, 1876, No. 39, p. 411.

—— *Neue Untersuchungen über den Einfluss der Ocular- und Spiegelstellung auf die Durchgangszeit; Bestimmung der persönlichen Gleichung.* Vierteljahrsschrift d. naturforsch. Gesells. (Zürich), XXI, 1876, 257. Also Wolf's Mittheilungen, V, 1879, No. 41, 1.

WUNDT: *Vorlesungen über die Menschen- und Thierseele.* Leipzig, 1863. Vol. I, pp. 38–40 and 366–371.

—— *Physiologische Psychologie.* 3d edition, Leipzig, 1887. Vol. II, pp. 261 ff., especially pp. 272–273 and 341–343.

ANON.: *Personal equation in observing circumpolars.* Science, VI, 1885, 449.